

Print Your Name

Student ID #

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| Problem      | Total Points | Score |
|--------------|--------------|-------|
| 1            | 8            |       |
| 2            | 7            |       |
| 3            | 6            |       |
| 4            | 7            |       |
| 5            | 7            |       |
| Part 2 Total | 35           |       |
| Part 1 Total | 35           |       |
| Exam Total   | 70           |       |

**Directions**

- Please check that your exam contains a total of 6 pages.
- Show all your work or you may not receive credit.
- Place a box around your **FINAL ANSWER** to each question.
- You may use one  $8.5 \times 11$  handwritten sheet of notes and a non-graphing calculator.
- If you use a trial and error (guess and check) or other numerical method when an algebraic method is available, you will not receive full credit.
- If you use an integral from a table other than those found on the back of this page, you will not receive full credit.

**Signature.** Please sign below to indicate that you have not and will not give or receive any unauthorized assistance on this exam.

Signature: \_\_\_\_\_

1. (8 total points)

(a) (6 points) Find the general solution to the following differential equation:

$$y' = (y + 1) \cos x.$$

Write your answer in the form  $y = f(x)$ .

(b) (2 points) Find the solution of the differential equation above that satisfies the initial condition  $y(0) = 1$ .

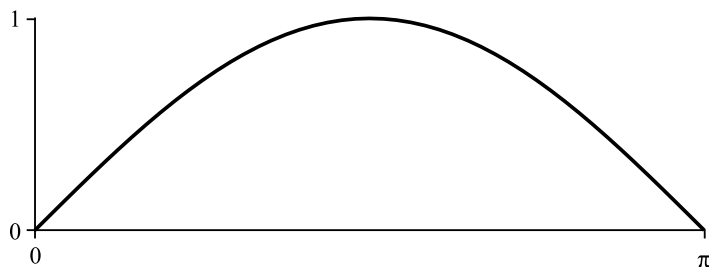
2. (7 points) A vat with 250 gallons of vinegar contains 5% acetic acid by volume. A more concentrated form of vinegar containing 8% acetic acid is poured into the vat at a rate of 5 gal/min. The mixture is stirred constantly and pours out at a rate of 5 gal/min.

Let  $V(t)$  be the volume of acetic acid in the mixture at a given time. SET UP a differential equation that describes the situation above. Include the initial condition. DO NOT SOLVE.

Differential Equation: \_\_\_\_\_

Initial Condition: \_\_\_\_\_

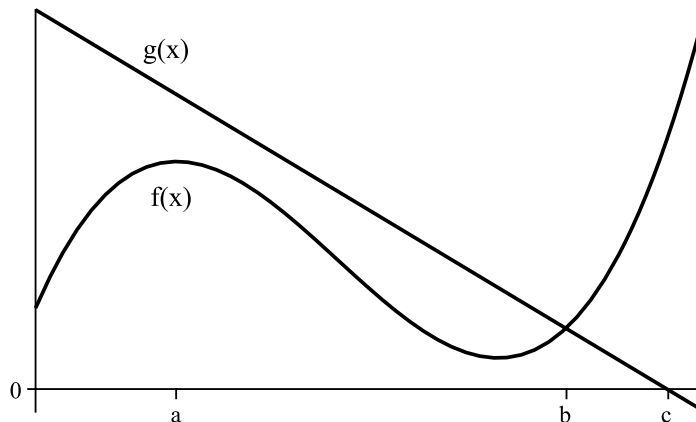
3. (6 points) Set up integrals to find the centroid  $(\bar{x}, \bar{y})$  of the region bounded by  $y = \sin x$ ,  $y = 0$ ,  $x = 0$ , and  $x = \pi$ . The area of the region is 2. DO NOT INTEGRATE.



$$\bar{x} = \underline{\hspace{15cm}}$$

$$\bar{y} = \underline{\hspace{15cm}}$$

4. (7 total points) Two functions,  $f(x)$ , and  $g(x)$  are graphed below.



Define

$$F(x) = \int_0^x f(t) dt \quad G(x) = \int_0^x g(t) dt.$$

- (a) (6 points) For each of the following, fill in the box with one of the following symbols:

< less than  
 = equal  
 > greater than

You do not need to show work.

$$F(b) \quad \square \quad G(b)$$

$$G(c) \quad \square \quad G'(c)$$

$$F'(b) \quad \square \quad G'(b)$$

$$F(a) \quad \square \quad F(b)$$

- (b) (1 point) What is  $F(F''(a))$ ?

5. (7 total points) The speed of sound in water is affected by the water's salinity, temperature, and depth. If I assume that the salinity and temperature do not change much with depth, the speed of sound in the Atlantic Ocean at a depth of  $x$  meters is approximately

$$v(x) = 1472 + 0.0163x \quad \text{meters per second.}$$

- (a) (3 points) Use the formula distance traveled = velocity  $\times$  time to determine the amount of time it takes the pulse to travel from a depth of  $x$  meters to a depth of  $x + \Delta x$  meters.

- (b) (2 points) Use the formula you found in part (a) to write the definite integral that gives the amount of time it takes a pulse of sound to travel from the surface of the ocean to a depth of 4000 meters. DO NOT EVALUATE.

- (c) (2 points) If you were to evaluate the integral in part (b), what units would your answer be in?