1. (a) Use $u = x - 1$, $du = dx$:

$$\int_1^3 (x-1)^{25}\,dx = \int_0^2 u^{25}\,du = \left.\frac{u^{26}}{26}\right]_0^2 = \frac{2^{26}}{26} = \frac{67108864}{26} = \boxed{\frac{33554432}{13}}.$$

(b) Integrate by parts with $\begin{array}{ll} u = t & dv = \sin t\,dt \\ du = dt & v = -\cos t \end{array}$:

$$\int t\sin t\,dt = -t\cos t + \int \cos t\,dt = \boxed{-t\cos t + \sin t + C.}$$

(c) Integrate by parts with $\begin{array}{ll} u = y & dv = e^{-y}\,dy \\ du = dy & v = -e^{-y} \end{array}$:

$$\int_0^1 ye^{-y}\,dy = \left[-ye^{-y}\right]_0^1 + \int_0^1 e^{-y}\,dy = \left[-ye^{-y} - e^{-y}\right]_0^1 = -e^{-1} - e^{-1} + 0 + 1 = \boxed{1 - 2e^{-1}.}$$

2. Solve $6x^3 - 31x^2 + 32x + 24 = -x^2 + 8x + 24$ and get $x = 0$, 1 or 4. These are the intersection points. The cubic is above from 0 to 1, and the quadratic is above from 1 to 4, so the area is

$$\begin{aligned} A &= \int_0^1 (6x^3 - 30x^2 + 24x)\,dx - \int_1^4 (6x^3 - 30x^2 + 24x)\,dx \\ &= \left[\frac{6}{4}x^4 - \frac{30}{3}x^3 + \frac{24}{2}x^2\right]_0^1 - \left[\frac{6}{4}x^4 - \frac{30}{3}x^3 + \frac{24}{2}x^2\right]_1^4 \\ &= \frac{7}{2} - 0 - \left(-64 - \frac{7}{2}\right) = \boxed{71.} \end{aligned}$$

3. We need to integrate with respect to x, so we use the shell method:

$$\begin{aligned} V &= \int_0^2 (2\pi x)\,\frac{1}{x}\sin\left(\frac{\pi}{2}x\right)\,dx = 2\pi\int_0^2 \sin\left(\frac{\pi}{2}x\right)\,dx \\ &= \left[\frac{-2\pi\cos\left(\frac{\pi}{2}x\right)}{\pi/2}\right]_0^2 = \left[-4\cos\left(\frac{\pi}{2}x\right)\right]_0^2 \\ &= -4\cos\pi + 4\cos 0 = 4 + 4 = \boxed{8.} \end{aligned}$$

4. (a) 0 mph

(b) $a(1/2) = v'(1/2) = \left(1 - \dfrac{\sin^2 \pi t}{\pi t}\right)\pi\Bigg|_{t=1/2} = \left(1 - \dfrac{\sin^2(\pi/2)}{\pi/2}\right)\pi = (1 - 2/\pi)\pi = \boxed{\pi - 2 \text{ mph/h.}}$

(c) Calculate $\sum f(x_i)\Delta x$ with sample points $\pi/2$, π, $3\pi/2$, and 2π. $\Delta x = \pi/2$:

$$\begin{aligned} S &= \left(1 - \frac{\sin^2(\pi/2)}{\pi/2}\right)\frac{\pi}{2} + \left(1 - \frac{\sin^2(\pi)}{\pi}\right)\frac{\pi}{2} + \left(1 - \frac{\sin^2(3\pi/2)}{3\pi/2}\right)\frac{\pi}{2} + \left(1 - \frac{\sin^2(3\pi)}{2\pi}\right)\frac{\pi}{2} \\ &= \left(1 - \frac{2}{\pi}\right)\frac{\pi}{2} + \frac{\pi}{2} + \left(1 - \frac{2}{3\pi}\right)\frac{\pi}{2} + \frac{\pi}{2} \\ &= \boxed{2\pi - 4/3 \approx 4.95 \text{ mph.}} \end{aligned}$$