Math 125A Summer 2009 **Name:** ____________________

Quiz #2

Show your work. Closed Notes.

1. (7 points) Evaluate the following integrals, if they exist.

(a) $\displaystyle\int_0^4 x^3\sqrt{x^2+9}\,dx$

Solution. We use the substitution method with $u = x^2 + 9$, which gives $du = 2x\,dx$. We compute the new limits to be 9 and 25. We also need to use the substitution $x^2 = u - 9$.

$$
\begin{aligned}
\int_0^4 x^3\sqrt{x^2+9}\,dx &= \frac{1}{2}\int_{x=0}^{x=4} x^2\sqrt{u}\,du \\
&= \frac{1}{2}\int_9^{25} (u-9)\sqrt{u}\,du \\
&= \frac{1}{2}\int_9^{25} \left(u^{3/2} - 9u^{1/2}\right)\,du \\
&= \frac{1}{2}\left[\frac{2}{5}u^{5/2} - 6u^{3/2}\right]_9^{25} \\
&= \frac{1}{2}\left[\frac{2}{5}\cdot 25^{5/2} - 6\cdot 25^{3/2} - \left(\frac{2}{5}\cdot 9^{5/2} - 6\cdot 9^{3/2}\right)\right] \\
&= \frac{1}{2}\left(1250 - 750 - \frac{486}{5} + 162\right) \\
&= \boxed{\frac{1412}{5} \text{ or } 282.4}
\end{aligned}
$$

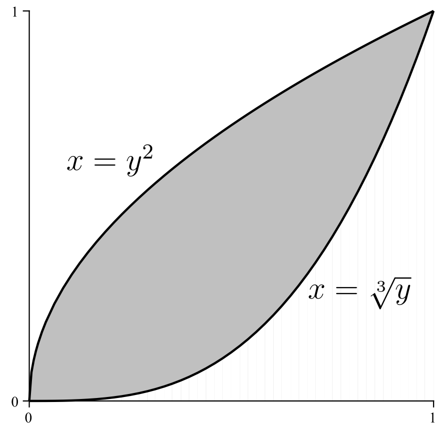
(b) $\displaystyle\int_{-3}^3 x^3\sqrt{x^2+9}\,dx$

Solution. The answer is $\boxed{0}$ since we are integrating an odd function from -3 to 3.

(Continued on back)

2. (8 points) Let R be the region enclosed by the functions $x = y^2$ and $y = x^3$. Determine the volume of the solid formed by rotating R around the y-axis.

Solution. Here is a picture of the region R:



Since the region is rotated around the y-axis, we compute the area of washers stacked vertically with outer radius $\sqrt[3]{y}$ and inner radius y^2. So the volume of the region is given by

$$\begin{aligned}\pi \int_0^1 \left[\left(\sqrt[3]{y}\right)^2 - \left(y^2\right)^2 \right] dy &= \pi \int_0^1 \left(y^{2/3} - y^4\right) dy \\ &= \pi \left[\frac{3}{5} y^{5/3} - \frac{1}{5} y^5 \right]_0^1 \\ &= \pi \left(\frac{3}{5} - \frac{1}{5} \right) \\ &= \boxed{\frac{2\pi}{5}}\end{aligned}$$

We could also do this using cylindrical shells. The shells have a height $\sqrt{x} - x^3$ and a circumference of $2\pi x$, so the volume of the region is given by

$$\begin{aligned}2\pi \int_0^1 x \left(\sqrt{x} - x^3\right) dx &= 2\pi \int_0^1 \left(x^{3/2} - x^4\right) dx \\ &= 2\pi \left[\frac{2}{5} x^{5/2} - \frac{1}{5} x^5 \right]_0^1 \\ &= 2\pi \left(\frac{2}{5} - \frac{1}{5} \right) \\ &= \boxed{\frac{2\pi}{5}}\end{aligned}$$