

### Quiz #3

Show your work. Closed Notes. You have 25 minutes.

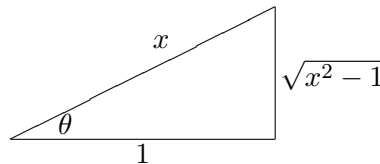
1. (4 points) Write down the equation that you would use to decompose the following rational function into partial fractions. DO NOT SIMPLIFY OR SOLVE.

$$\frac{1}{x^3(x+5)^2(x^2+1)}$$

Answer:  $\frac{A}{x^3} + \frac{B}{x^2} + \frac{C}{x} + \frac{D}{(x+5)^2} + \frac{E}{x+5} + \frac{Fx+G}{x^2+1}$

2. (4 points) Write  $\sin\left(\cos^{-1}\left(\frac{1}{x}\right)\right)$  in a form that does not involve trigonometric functions.

Answer: Let  $\theta = \cos^{-1}\left(\frac{1}{x}\right)$ , then  $\sin \theta = \boxed{\frac{\sqrt{x^2-1}}{x}}$ .



3. (7 points) Give the general form of  $\int \frac{x^4+1}{x^3-x^2} dx$ .

Answer: First we use polynomial long division:

$$\begin{array}{r} x+1 \\ x^3-x^2 \overline{) x^4 \phantom{+x^3} +1} \\ \underline{-x^4+x^3} \phantom{+1} \\ x^3 \phantom{+x^2} +1 \\ \underline{-x^3+x^2} \\ x^2+1 \end{array}$$

So we get  $\frac{x^4+1}{x^3-x^2} = x+1 + \frac{x^2+1}{x^2(x-1)}$ .

Then we use partial fractions:

$$\begin{aligned} \frac{x^2+1}{x^2(x-1)} &= \frac{A}{x} + \frac{B}{x^2} + \frac{C}{x-1} \\ x^2+1 &= Ax(x-1) + B(x-1) + Cx^2 \\ x^2+1 &= (Ax^2 + Cx^2) + (-Ax + Bx) - B \end{aligned}$$

This gives us the equations

$$A + C = 1 \qquad -A + B = 0 \qquad -B = 1$$

So  $B = -1, A = -1$ , and  $C = 2$ . So the integral that we want is

$$\int \left( x+1 - \frac{1}{x} - \frac{1}{x^2} + \frac{2}{x-1} \right) dx = \frac{x^2}{2} + x - \ln|x| + \frac{1}{x} + 2\ln|x-1| + K$$