Math 125A Summer 2009 **Name:** ____________________

Quiz #4

Show your work. Closed Notes. You have 25 minutes.

1. (6 total points) For this problem, round your answers to 4 decimal places.

   (a) (4 points) Use Simpson's rule and $n = 4$ to estimate $\int_0^2 e^{-x^2}\, dx$.

   Solution: Note $\Delta x = 1/2$. Then we have

$$\frac{1/2}{3}\left[e^0 + 4e^{-(1/2)^2} + 2e^{-1^2} + 4e^{-(3/2)^2} + e^{-2^2}\right] = \frac{1}{6}\left(1 + 4e^{-1/4} + 2e^{-1} + 4e^{-9/4} + e^{-4}\right)$$
$$\approx \boxed{0.8818\,.}$$

   (b) (2 points) The actual value of the integral (rounded to 4 decimal places) is 0.8821. What is the error in your approximation above?

$$\textit{Solution: } \text{Actual} - \text{Approximate} = 0.8821 - 0.8818 = \boxed{0.0003\,.}$$

2. (4 points) Set up an integral to give the length of the arc given by the function

$$y = x^3 - 6x^2 + 8x$$

   from $(0, 0)$ to $(5, 15)$. DO NOT SOLVE.
   Solution: We calculate $y' = 3x^2 - 12x + 8$. Then the arc length is

$$\boxed{\int_0^5 \sqrt{1 + (3x^2 - 12x + 8)^2}\, dx.}$$

3. (5 points) The following improper integral converges. Determine its value.

$$\int_1^\infty e^{1-x}\, dx.$$

   Solution: Let $u = 1 - x$. Then $du = -dx$, so we have

$$\int e^{1-x}\, dx = -\int e^u\, du = -e^u = -e^{1-x}\,.$$

   Therefore,

$$\int_1^\infty e^{1-x}\, dx = \lim_{t\to\infty}\left(-e^{1-x}\Big|_1^t\right) = \lim_{t\to\infty}\left(-e^{1-t} + e^0\right) = 0 + 1 = \boxed{1.}$$