SOLUTIONS TO QUIZ 2

(1) (a) $x = t^2, y = t^3$ is on the bottom right because x is always positive.
(b) $x = \sin t, y = \cos(3t)$ is on the top right because if $x = 0$ then $t = 0$ or π, in which case y is 1 or -1.
(c) $x = t\cos(2t), y = t\sin(2t)$ is on the top left because the sin and cos makes it go around in a circle, and multiplying by t makes the radius grow as the angle increases.
(d) $x = t, y = t^3$ gives the curve $y = x^3$, which is in the bottom left.

(2)

$$\mathbf{v}(t) = \mathbf{r}'(t) = \langle 3t^2 + 1, 2t + 1, 1\rangle$$
$$\mathbf{a}(t) = \mathbf{v}'(t) = \mathbf{r}''(t) = \langle 6t, 2, 0\rangle$$
$$\mathbf{F}(t) = m\mathbf{a}(t) = \boxed{\langle 6tm, 2m, 0\rangle}$$

(3) As x goes from 0 to $1/\sqrt{2}$, t goes from 0 to $\pi/4$. So we have

$$A = \int_0^{1/\sqrt{2}} y\,dx = \int_0^{\pi/4} \frac{1}{\cos t}\cos t\,dt = \int_0^{\pi/4} 1\,dt = \boxed{\pi/4}$$

(4) (a) We compute

$$\mathbf{r}'(t) = \langle 2\cos t, 4, -2\sin t\rangle$$
$$|\mathbf{r}'(t)| = \sqrt{4\cos^2 t + 16 + 4\sin^2 t} = \sqrt{20} = 2\sqrt{5},$$

so

$$\mathbf{T}(t) = \frac{\mathbf{r}(t)}{|\mathbf{r}'(t)|} = \left\langle \frac{\cos t}{\sqrt{5}}, \frac{2}{\sqrt{5}}, -\frac{\sin t}{\sqrt{5}}\right\rangle.$$

(b) We compute

$$\mathbf{T}'(t) = \left\langle \frac{-\sin t}{\sqrt{5}}, 0, \frac{-\cos t}{\sqrt{5}}\right\rangle$$
$$|\mathbf{T}'(t)| = \sqrt{\frac{\sin^2 t + \cos^2 t}{5}} = \frac{1}{\sqrt{5}},$$

so

$$\mathbf{N}(t) = \frac{\mathbf{T}'(t)}{|\mathbf{T}'(t)|} = \langle \sin t, 0, -\cos t\rangle.$$

(c) We have

$$\kappa(t) = \frac{|\mathbf{T}'(t)|}{|\mathbf{r}'(t)|} = \frac{1/\sqrt{5}}{2\sqrt{5}} = \frac{1}{10}$$