Directions. Show your work and write complete solutions or you may not receive credit. If you need more room, use the backs of the pages and indicate to the reader that you have done so.

1. (12 total points; 3 points each) Let $\mathbf{a} = 2\mathbf{i} - 4\mathbf{j} + 4\mathbf{k}$ and $\mathbf{b} = \mathbf{i} - \mathbf{j}$.

   (a) Compute $\mathbf{a} \times \mathbf{b}$.

$$\begin{vmatrix} i & j & k \\ 2 & -4 & 4 \\ 1 & -1 & 0 \end{vmatrix} \begin{matrix} i & j \\ 2 & -4 \\ 1 & -1 \end{matrix}$$

$$\begin{aligned} &0i + 4j - 2k \\ &+4i + 0j + 4k = \boxed{4i + 4j + 2k} \end{aligned}$$

   (b) Find the angle between $\mathbf{a}$ and $\mathbf{b}$.

$$\cos\theta = \frac{a \cdot b}{|a||b|} = \frac{2+4+0}{6\sqrt{2}} = \frac{6}{6\sqrt{2}} = \frac{1}{\sqrt{2}} = \frac{\sqrt{2}}{2}$$

$$|a| = \sqrt{2^2 + (-4)^2 + 4^2} = \sqrt{4+16+16} = \sqrt{36} = 6$$

$$|b| = \sqrt{1 + (-1)^2} = \sqrt{2}$$

so $\theta =$ $\boxed{\pi/4 \text{ or } 45^\circ}$

   (c) Find a vector of length 2 that points in the same direction as $\mathbf{a}$.

$|a| = 6$, so $\left|\frac{a}{3}\right| = 2$. $\frac{a}{3} = \boxed{\left\langle \frac{2}{3}, -\frac{4}{3}, \frac{4}{3} \right\rangle}$

   (d) Find the projection of $\mathbf{a}$ onto $\mathbf{b}$ ($\text{proj}_{\mathbf{b}}\mathbf{a}$).

$$\text{proj}_b\, a = \frac{a \cdot b}{b \cdot b} b = \frac{6}{2}\langle 1, -1, 0 \rangle$$

$$= \boxed{\langle 3, -3, 0 \rangle}$$

2. (6 total points; 3 points each)

(a) Find the equation of a line perpendicular to the plane $x - y + 2z = 12$ and passing through the origin.

$\vec{r} = \vec{r}_0 + \vec{v}t$

$\vec{r}_0 = \langle 0,0,0 \rangle$

$\vec{v} = \langle 1,-1,2 \rangle$

so $\boxed{\vec{r} = \langle t, -t, 2t \rangle}$

or $x = t$, $y = -t$, $z = 2t$.

(b) Find the point where the plane in part (a) meets the line you found in part (a).

A point on $\langle t, -t, 2t \rangle$ is on $x - y + 2z = 12$ if

$t - (-t) + 2(2t) = 12$

$t + t + 4t = 12$

$6t = 12$

$t = 2$

so the point is $\boxed{(2, -2, 4)}$

3. (6 points) Find the equation of a plane that passes through the points (1, 0, 0), (2, 0, 1), (3, 0, 2), and (2, 1, 2), if such a plane exists.

A B C D

$\vec{AB} = \langle 1,0,1 \rangle$
$\vec{AC} = \langle 2,0,2 \rangle$) parallel.
$\vec{AD} = \langle 1,1,2 \rangle$

Normal vector $n =$
$\langle 1,0,1 \rangle \times \langle 1,1,2 \rangle = \langle -1,-1,1 \rangle$

so the equation is (using point A)

$-1(x-1) + -1(y-0) + 1(z-0) = 0$

$-x + 1 - y + z = 0$

$\boxed{-x - y + z + 1 = 0}$

You can check that all four points are on here.