

1. The values of  $f$  at the midpoints are approximately, beginning at the top left:

$$5, 5, 10, 4, 4, 5, 3, 3, 5.$$

Adding these together, we get 44; multiplying by  $\Delta x \Delta y = 4$ , we get a volume under the graph of 176. Dividing by the area of the domain, which is 36, we get 4.89. Your estimate may be slightly different. The actual value is 4.75.

2. Integrate first with respect to  $x$  and then  $y$  to make it easier:

$$\begin{aligned} \int_0^\pi \int_0^3 y \cos(xy) \, dx dy &= \int_0^\pi \left[ \sin(xy) \right]_{x=0}^{x=3} dy \\ &= \int_0^\pi \sin(3y) \, dy \\ &= -\frac{1}{3} \cos(3y) \Big|_0^\pi \\ &= -\frac{1}{3}(-1) + \frac{1}{3}(1) \\ &= \frac{2}{3}. \end{aligned}$$

3. The easiest thing is to integrate  $y$  from 0 to 1 and  $x$  from  $-1$  to  $\sqrt{y}$ , although you could also split it into two parts and integrate over the square in the second quadrant and then  $x$  from 0 to 1 and  $y$  from  $x^2$  to 1. I will do it the first way.

$$\begin{aligned} \int_0^1 \int_{-1}^{\sqrt{y}} 2x \, dx dy &= \int_0^1 \left[ x^2 \right]_{x=-1}^{x=\sqrt{y}} dy \\ &= \int_0^1 [y - 1] \, dy \\ &= \frac{1}{2}y^2 - y \Big|_0^1 \\ &= \frac{1}{2} - 1 \\ &= -\frac{1}{2} \end{aligned}$$