Worksheet 6 — Math 126 — Summer 2010

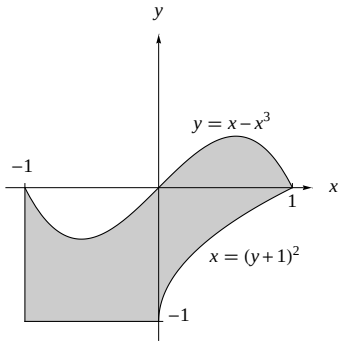
The purpose of this worksheet is to give you some practice doing double integrals. They will definitely not be easy. Good luck!

1. Find the volume under the surface $z = 2x + y^2$ and above the region in the first quadrant of the xy plane that is bounded by $y = x^2$ and $y = x^3$.

Answer: $4/35$

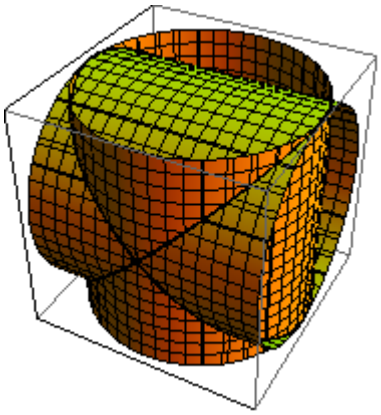
2. Let R be the following shaded region to the right. Compute the following integral: [Hint: you will have to split the region into at least two peices]

$$\iint_R 2x\, dA$$

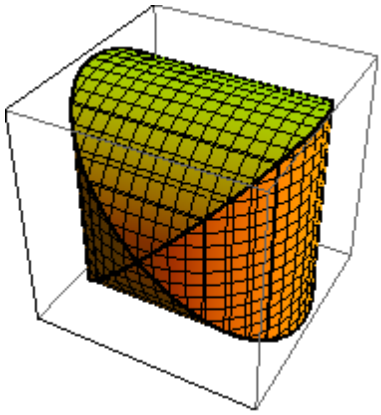


Answer: $-4/15$

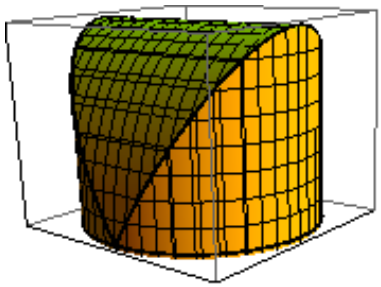
3. Find the volume of the space bounded by the two cylinders $x^2+y^2=1$ and $y^2+z^2=1$. [Hints:
   (a) Imagine the second cylinder as a tunnel going over the top of you. What is the equation for the height of the tunnel?.
   (b) Integrate the height of the tunnel over the region $x^2+y^2=1$ to get the volume between the tunnel and above the "ground". Since the region is symmetric, we multiply that answer by 2 to get the final answer.
   (c) You have the choice to integrate with respect to y and then x or x first then y. One of these makes the integral really difficult, so do it the easy way.]



The two cylinders



The intersection



The top half of the intersection

Answer: rhymes with "mixed green nerds"