ANSWERS TO WORKSHEET 2 (POLAR COORDINATES)

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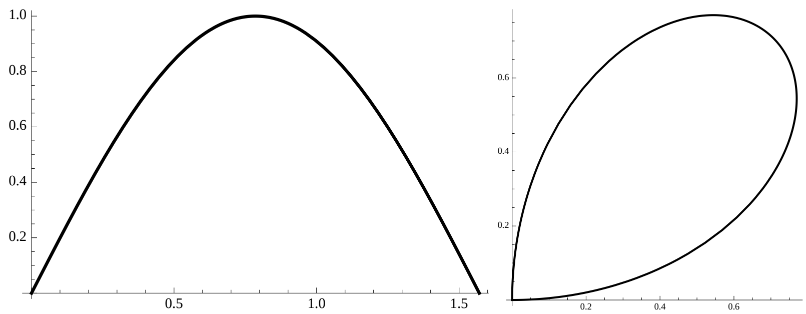
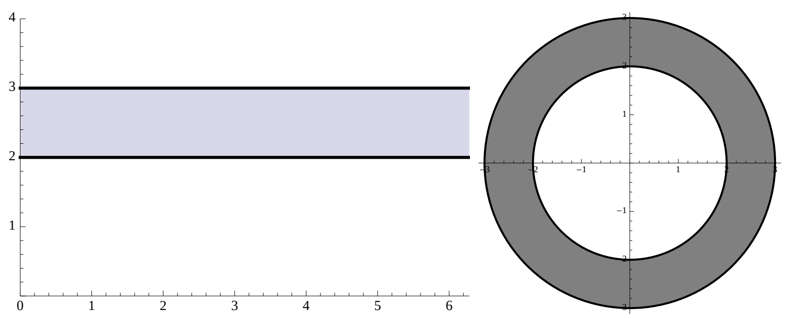
(1a)



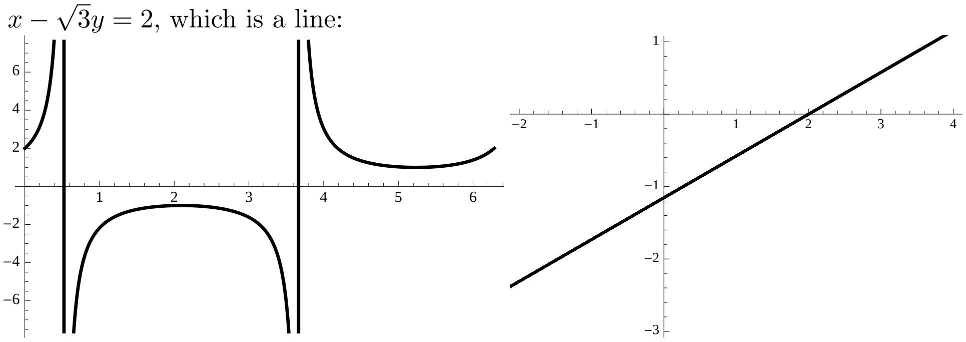
(1b)



(1c) The math on this one is pretty tricky. The main idea is to eliminate the r and θ in the equation. I can multiply both sides of $\sec(\theta + \pi/3) = r$ by $\cos(\theta + \pi/3)$ to get

$$\begin{aligned} 1 &= r\cos(\theta + \pi/3) = r(\cos\theta\cos\pi/3 - \sin\theta\sin\pi/3) \\ &= r\left(\left(\frac{x}{r}\right)\left(\frac{1}{2}\right) - \left(\frac{y}{r}\right)\left(\frac{\sqrt{3}}{2}\right)\right) = \frac{x}{2} - \frac{\sqrt{3}y}{2} \end{aligned}$$

So $x - \sqrt{3}y = 2$, which is a line:



(2a) x^2+y^2 is obviously 4, so this is a circle, which has polar equation $r = 2$. The square root sign means to take the positive square root only, so technically x should always be positive, which means we have the right half of a circle. This doesn't change the polar equation, but it means that $-\pi/2 \le \theta \le \pi/2$.

(2b) This takes some ingenuity, but to get the line $x + y = 1$ we need $r\cos\theta + r\sin\theta = 1$. Since $\cos(\pi/4) = \sin(\pi/4) = 1/\sqrt{2}$, we can rewrite this as

$$1 = r\sqrt{2}\,(\cos\theta\sin(\pi/4) + \sin\theta\cos(\pi/4)) = r\sqrt{2}\sin(\theta + \pi/4)$$

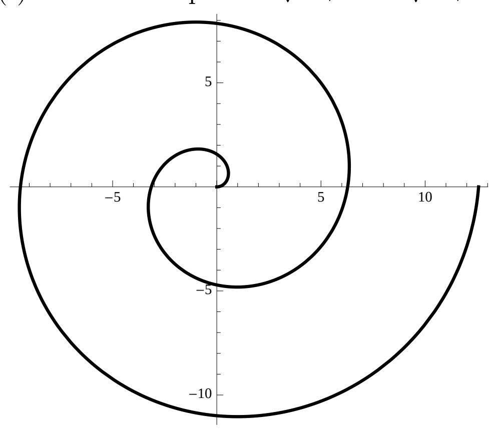
Solving for r, we get

$$r = \frac{1}{\sqrt{2}\sin(\theta + \pi/4)} = \frac{\csc(\theta + \pi/4)}{\sqrt{2}}.$$

So the equations for the region are $0 \le \theta \le \pi/2$ and $0 \le r \le \frac{\csc(\theta+\pi/4)}{\sqrt{2}}$.

(3) The velocity vector is $\langle r'\cos\theta - r\sin(\theta)\theta', r'\sin\theta + r\cos(\theta)\theta'\rangle$. When we take the length of this vector lots of sin's and cos's cancel out to give us $\sqrt{[r'(t)]^2 + [r(t)]^2[\theta'(t)]^2}$.

(4a) $r'(t) = 1$ and $\theta'(t) = 1$. So the speed is $\sqrt{1 + r^2} = \sqrt{1 + \theta^2} = \sqrt{1 + t^2}$.



(4b) $r'(t) = \cos t$ and $\theta'(t) = 1$. So the speed is $\sqrt{\cos^2 t + \sin^2 t} = 1$.