Worksheet 2 — Math 126 — Summer 2010

The goal of this worksheet is to get familiar with the use of polar coordinates and to practice the conversion from polar coordinates to Cartesian and vice versa. You should observe that some regions are easier to understand in Cartesian coordinates whereas for others the choice of polar coordinates is much more suitable. The skill of being able to chose the right coordinate system will be invaluable when we start evaluating double integrals as in chapter 15. Later on you may wish to compare the regions you sketch today with the ones in problems 1-6, Section 15.4.

The following trig identity will be useful for the worksheet:

$$\sin(\alpha+\beta) = \sin(\alpha)\cos(\beta) + \cos(\alpha)\sin(\beta)$$

$$\cos(\alpha+\beta) = \cos(\alpha)\cos(\beta) - \sin(\alpha)\sin(\beta)$$

And here are the conversion formulas

$$x = r\cos\theta, \quad y = r\sin\theta$$
$$r^2 = x^2 + y^2, \quad \tan\theta = \frac{y}{x}$$

1. Describe each curve (region) below in Cartesian coordinates. Then sketch the curve (region). Indicate which coordinate system you used for sketching.
   (a) $r = \sin 2\theta$, $0 \le \theta \le \frac{\pi}{2}$
   (b) $r^2 - 5r + 6 = (r-3)(r-2) < 0$
   (c) $r = \sec(\theta + \pi/3)$. Simplify your expression in Cartesian coordinates as much as possible, using trig identities.

2. Describe each curve (region) below in polar coordinates. Then sketch the curve (region). Indicate which coordinate system you used for sketching.
   (a) $x = \sqrt{4 - y^2}$
   (b) $x \ge 0$, $y \ge 0$, $x + y \le 1$. Simplify your polar coordinate expression for $x + y$ to something involving only one trig function. Hint: compare to 1(c).

3. Suppose the position of a particle as a function of time is given in polar coordinates as $(r(t),\ \theta(t))$. Find the speed of the particle, that is, the magnitude of the velocity vector, in terms of r, θ, and their derivatives. Hint: One way to do this is to convert the position to Cartesian coordinates, use the chain rule to find the velocity, and then the speed, then convert the result back to polar coordinates.

4. Suppose the particle moves along one of the following curves so that $\theta = t$:

   (a) $r = \theta$

   (b) $r = \sin\theta$

   Sketch the curves and find the speed of the particle in each case. Before you do the calculuations, guess whether you think the particle will move with constant speed, and if not, where on the curve it will move faster and where slower. Explain briefly the reasons for your guess. After you do the calculations, note whether your guess was correct.

5. (*Equations of lines in polar coordinates.*) If you have time left during the section, or later at home, consider the following generalization of some of the ideas introduced above.

   Any line on the plane can be described by the equation

   $$ax + by = d$$

   Rewrite the equation in polar coordinates. Using the insights you gained in 1(c) and 2(b) (and trig identity!), simplify your answer to have the form

   $$r = C\sec(\theta_0 - \theta)$$

   What is the geometric meaning of the angle θ_0 in your final formula?