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Score

1. Evaulate the iterated integral by converting to polar coordinates.

$$\int_{-3}^{3}\int_{0}^{\sqrt{9-y^2}} \sin(x^2+y^2)\,dx\,dy$$

Solution: The region is the right-half of a circle of radius 3, so r goes from 0 to 3 and θ goes from $-\pi/2$ to $\pi/2$. The limits do not depend on each other, so it does not matter which order we do the integration. So we have

$$\begin{aligned}
\int_0^3\int_{-\pi/2}^{\pi/2} \sin(r^2)r\,d\theta\,dr &= \int_0^3 \Big[\sin(r^2)r\theta\Big]_{\theta=-\pi/2}^{\theta=\pi/2} dr \\
&= \int_0^3 \pi\sin(r^2)r\,dr \\
[u=r^2, du=2rdr] \quad &= \frac{1}{2}\pi\int_{u=0}^{u=9}\sin u\,du \\
&= -\frac{\pi}{2}\Big[\cos u\Big]_0^9 \\
&= \frac{\pi}{2}\left(1-\cos 9\right).
\end{aligned}$$