Print your name: _____

Score

1. Suppose that X and Y are random variables with joint density function

$$f(x,y) = \begin{cases} 4xy & \text{if } 0 \le x \le 1, 0 \le y \le 1 \\ 0 & \text{otherwise} \end{cases}$$

Determine the probability that $X^2 + Y^2$ is bigger than 1.

Solution: We calculate

$$\begin{aligned} \int_0^1 \int_{\sqrt{1-x^2}}^1 4xy \, dy \, dx &= \int_0^1 2xy^2 \Big|_{y=\sqrt{1-x^2}}^{y=1} \, dx \\ &= \int_0^1 2x - 2x(1-x^2) \, dx \\ &= \int_0^1 2x^3 \, dx \\ &= \frac{1}{2}x^4 \Big|_0^1 \\ &= \frac{1}{2}. \end{aligned}$$