Print your name: ____________________

Score

1. Determine $\int_C y^2\,dx + 2y(x+1)\,dy$, where C is the curve parametrized by

$$x(t) = (-t^2/2)\cos t, \qquad y(t) = \sin^2(t)/2 + t, \qquad 0 \le t \le 3\pi/2.$$

Hint: First check to see if the vector field is conservative.

Solution: First, notice that $\frac{\partial P}{\partial y} = \frac{\partial Q}{\partial x} = 2y$, and the vector field is defined at all points, so the vector field is conservative. At this point there are two ways to proceed.

1. Find the antigradient and use the fundamental theorem of vector calculus. We have $\int P\,dx = y^2x$ and $\int Q\,dy = xy^2 + y^2$, so the antigradient is $f = y^2 + xy^2$. The endpoints of the curve are $(0,0)$ and $(0, 3\pi/2 + 1/2)$, so the value of the integral is
$$y^2 + xy^2\Big|_{(0,0)}^{(0,3\pi/2+1/2)} = (3\pi/2 + 1/2)^2.$$
2. Since the vector field is path independent, choose another path with the same endpoints that is simpler. I would choose the straight line segment from $(0,0)$ to $(0, 3\pi/2+1/2)$, which is parametrized by $\mathbf{r}(t) = \langle 0, t\rangle$ for $0 \le t \le 3\pi/2+1/2$. We have
$$\int_0^{3\pi/2+1/2} 2t\,dt = (3\pi/2 + 1/2)^2.$$