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Score

1. Parametrize the torus (doughnut) given in spherical coordinates by the equation

$$\rho = \sin \phi.$$

Solution:

$$\mathbf{r}(\phi, \theta) = \langle \sin^2 \phi \sin \theta, \sin^2 \phi \cos \theta, \sin \phi \cos \phi \rangle$$

2. Find a unit normal vector at the point $(1, 0, 0)$, which is when $\rho = 1, \phi = \pi/2, \theta = 0$.

Solution:

$$\mathbf{r}_\phi = \langle 2 \sin \phi \cos \phi \sin \theta, 2 \sin \phi \cos \phi \cos \theta, \cos^2 \phi - \sin^2 \phi \rangle = \langle 0, 0, -1 \rangle$$

$$\mathbf{r}_\theta = \langle \sin^2 \phi \cos \theta, -\sin^2 \phi \sin \theta, 0 \rangle = \langle 1, 0, 0 \rangle$$

The cross product is $\langle 0, -1, 0 \rangle$. It is already a unit vector.

You could also put $\langle 0, 1, 0 \rangle$ which is the natural orientation because it points out of the torus.