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Score

1. Use Stokes' Theorem to evaluate

$$\int_C \langle yz + 3xz, 5x + z + xz, 6 + xy + e^{xyz} \rangle \cdot d\mathbf{r},$$

where C is the circle at the intersection of the cylinder $x^2 + y^2 = 4$ and the plane $z = 14$, oriented in the counterclockwise direction when looking down from above.

Solution: Let $\mathbf{F}$ be the vector field that we are integrating. Notice that $\operatorname{curl}\mathbf{F} = \langle xze^{xyz} - 1, 3x + yze^{xyz}, 5\rangle$. For Stokes' Theorem, we can use any surface that has the given circle as boundary, but I would choose the disk of radius 2 inside the plane $z = 14$. This can be parametrized by $\mathbf{r}(x, y) = \langle x, y, 14\rangle$ for (x, y) in the disk of radius 2, which I will call D. We calculate $\mathbf{r}_x \times \mathbf{r}_y = \langle 0, 0, 1\rangle$. So by Stokes' Theorem, the integral above is the same as

$$\iint_D (\operatorname{curl}\mathbf{F}) \cdot \langle 0, 0, 1\rangle \, dA = \iint_D 5 \, dA = 5(\text{area of disk}) = 20\pi.$$