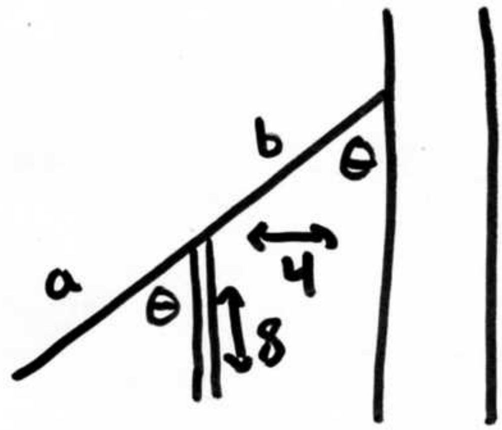
Solutions For Final Review

1.



length = a + b

$a = \frac{8}{\cos\theta}$ $b = \frac{4}{\sin\theta}$

$$l = \frac{8}{\cos\theta} + \frac{4}{\sin\theta}$$

$$l' = \frac{8\sin\theta}{\cos^2\theta} - \frac{4\cos\theta}{\sin^2\theta} = \frac{8\sin^3\theta - 4\cos^3\theta}{\sin^2\theta\cos^2\theta} = 0$$

So $8\sin^3\theta - 4\cos^3\theta = 0$

So $\frac{\sin^3\theta}{\cos^3\theta} = \frac{1}{2}$, so $(\tan\theta)^3 = \frac{1}{2}$, so $\tan\theta = \sqrt[3]{\frac{1}{2}}$

so $\theta = \tan^{-1}\left(\sqrt[3]{\frac{1}{2}}\right) \approx 0.6708$

$l'(\pi/6) < 0$ $\quad \overset{.6708}{-\;|\;+} \quad$ $l'(\pi/4) > 0$

So this is a minimum.

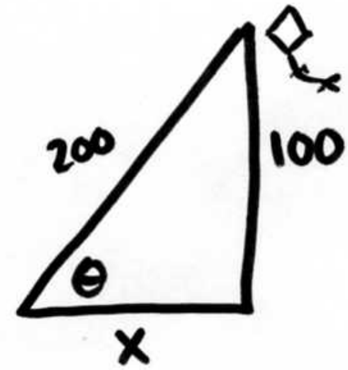
$$l(.6708) = \boxed{16.65}$$

(2) $x = 200\cos\theta$

$\frac{dx}{dt} = 8$

$\frac{d\theta}{dt} = ??$

$\sin\theta = \frac{1}{2}$



$$\frac{dx}{dt} = -200\sin\theta\,\frac{d\theta}{dt}$$

$$8 = (-200)\left(\tfrac{1}{2}\right)\frac{d\theta}{dt}$$

$$\frac{d\theta}{dt} = -\frac{8}{100} = \boxed{\frac{-2}{25}}$$

So it is decreasing at $2/25$ rad/s.

(3) Equation: $y = 5^x$

$x_1 = 1 \quad y_1 = 5$

$\Delta x = -.01 \quad \Delta y = ??$

$x_2 = .99 \quad y_2 = ??$

$y' = 5^x \ln 5$

$y'(1) = 5\ln 5$

$$\Delta y \approx y'(1)\,\Delta x = (5\ln 5)(-.01) = -.0805$$

So $y_2 = 5 - .0805 = 4.9195$.

(compare to actual value $5^{.99} = 4.9202$.)

(4) • $\lim_{x\to 1^+} F(x)$: since $x-1>0$, $|x-1| = x-1$.

So $F(x) = \dfrac{x^2-1}{x-1} = \dfrac{(x+1)(x-1)}{x-1} = x+1$

so $\lim_{x\to 1^+} F(x) = 1+1 = \boxed{2}$

• $\lim_{x\to 1^-} F(x)$: since $x-1<0$, $|x-1| = -(x-1)$.

So $F(x) = \dfrac{x^2-1}{-(x-1)} = -(x+1) = \boxed{-2}$

Since the one-sided limits are different, $\lim_{x\to 1} F(x)$ does not exist.

(5) Method 1:

$x^x = e^{x\ln x}$

$\frac{d}{dx}(x^x) = (e^{x\ln x})(\ln x + 1)$

$= x^x(\ln x + 1)$

Method 2:

$y = x^x$

$\ln y = x \ln x$

$\frac{y'}{y} = \ln x + 1$

$y' = x^x(\ln x + 1)$

(6) $f(x)$ is squeezed between x^2 and 0

$0 \leq f(x) \leq x^2$ for all x.

At 0, $\lim_{x\to 0} 0 = 0$ and $\lim_{x\to 0} x^2 = 0$, so

$\lim_{x\to 0} f(x) = 0$.