Solutions to Review Session

1) Variables: c = cost r = radius of ball.

Equation: $c = r^3 + 2r^2$

$c_1 = 16$ = current cost $r_1 = 2$ = current radius

$c_{true} = 15$ = true cost r_{true} = true radius = $r_1 + \Delta r$ (find this)

$\Delta c = -1$ = difference Δr = difference

Derivative: $\frac{dc}{dr} = 3r^2 + 4r$. Evaluate derivative at r_1: $3(2)^2 + 4(2) = 20$.

Tangent Line Approximation

$\Delta c \approx \frac{dc}{dr} \Delta r$

$\Delta c \approx 20 \Delta r$

$-1 \approx 20 \Delta r$

$-1/20 \approx \Delta r$

so $r_{true} = r_1 + \Delta r \approx 2 + (-1/20)$

$\approx \boxed{1.95\text{ cm}}$

2) Variables:

V = volume r = radius = 5 h = height = 6

$\frac{dV}{dt} = 10\pi$ $\frac{dr}{dt}$ = (find this) $\frac{dh}{dt} = 1$

Equation: $V = \frac{\pi}{3} r^2 h$

Derivative with respect to time (use product rule)

$$\frac{dV}{dt} = \frac{2\pi}{3} r \frac{dr}{dt} h + \frac{\pi}{3} r^2 \frac{dh}{dt}$$

$$10\pi = \frac{2\pi}{3}(5)\left(\frac{dr}{dt}\right)(6) + \frac{\pi}{3}(5)^2(1)$$

$$10\pi = 20\pi\left(\frac{dr}{dt}\right) + \frac{25\pi}{3}$$

$$\frac{5}{3}\pi = 20\pi \frac{dr}{dt}$$

$$\boxed{\frac{1}{12} = \frac{dr}{dt}}$$

③ Variables: F = flow r = radius

Equation: $F = kr^4$

F_1 = current flow $= kr_1^4$ r_1 = current radius

F_{true} = new flow r_{true} = new radius

ΔF = change in flow Δr = change in radius

Remember: $\frac{\Delta F}{F_1}$ is % change in flow; $\frac{\Delta r}{r_1}$ is % change in radius.
(find this)   (3%)

$$\frac{dF}{dr} = 4kr^3 \quad \text{evaluate at } r_1 \text{ and we have } 4kr_1^3$$

Tangent line approx:

$$\Delta F \approx \frac{dF}{dr}\Delta r$$

$$\Delta F \approx 4kr_1^3 \Delta r$$

$$\frac{\Delta F}{F_1} \approx \frac{4kr_1^3\Delta r}{F_1} = \frac{4kr_1^3\Delta r}{kr_1^4}$$

$$\frac{\Delta F}{F_1} \approx 4\left(\frac{\Delta r}{r_1}\right) = 4(3\%) = \boxed{12\%}$$

④ Variables:

V = Volume h = height = 240 s = side of top square = 375 A = area of top square = s^2 = 140625

$\frac{dV}{dt}$ = (find this) $\frac{dh}{dt} = 2$ $\frac{ds}{dt} = -\frac{25}{8}$ $\frac{dA}{dt} = -2343.75$

$B = 480^2$ = constant

Equations:

$$V = \frac{1}{3}h\left(A + B + \sqrt{AB}\right) \qquad \frac{dV}{dt} = \frac{1}{3}\left(\frac{dh}{dt}\right)\left(A+B+\sqrt{AB}\right) + \frac{1}{3}h\left(\frac{dA}{dt} + \frac{B\frac{dA}{dt}}{2\sqrt{AB}}\right)$$

$$s = \frac{25}{16}(480 - h) \qquad \frac{ds}{dt} = -\frac{25}{16}\frac{dh}{dt}$$

$$A = s^2 \qquad \frac{dA}{dt} = 2s\frac{ds}{dt}$$

Before we can find $\frac{dV}{dt}$, we need to find A, S, $\frac{dA}{dt}$, and $\frac{ds}{dt}$.

$$s = \frac{25}{16}(480 - 240) = 375$$

$$A = s^2 = 140625$$

$$\frac{ds}{dt} = -\frac{25}{16}\frac{dh}{dt} = -\frac{25}{16}(2) = -\frac{25}{8}$$

$$\frac{dA}{dt} = 2s\frac{ds}{dt} = 2(375)\left(-\frac{25}{8}\right) = -2343.75$$

$$\frac{dV}{dt} = \frac{1}{3}(2)\left(140625 + 230400 + \sqrt{(140625)(230400)}\right) + \frac{1}{3}(240)\left(-2343.75 + \frac{(230400)(-2343.75)}{2\sqrt{(140625)(230400)}}\right)$$

$= 59850$ ft³/year

$\boxed{\text{OR } 16625 \text{ blocks}}$

⑤ $$2\sin\left[\ln(\cos(e^x)) + \tan^{-1}(2x)\sqrt{52x}\right] \cdot \cos\left[\ln(\cos(e^x)) + \tan^{-1}(2x)\sqrt{52x}\right]$$

$$\cdot \left(\frac{1}{\cos(e^x)}(-\sin(e^x))(e^x) + \frac{1}{1+(2x)^2}(2)\sqrt{52x} + \tan^{-1}(2x)\frac{1}{2\sqrt{52x}}(52)\right)$$