Review of trig functions

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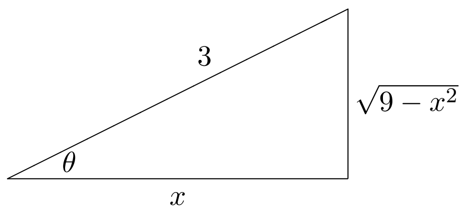
Trig function basics.

$$\sin\theta = \frac{\text{opposite}}{\text{hypoteneuse}} \qquad \sec\theta = \frac{1}{\cos\theta} \qquad \sec^{-1} x = \cos^{-1}(1/x)$$
$$\cos\theta = \frac{\text{adjacent}}{\text{hypoteneuse}} \qquad \csc\theta = \frac{1}{\sin\theta} \qquad \csc^{-1} x = \sin^{-1}(1/x)$$
$$\tan\theta = \frac{\text{opposite}}{\text{adjacent}} \qquad \cot\theta = \frac{1}{\tan\theta} \qquad \cot^{-1} x = \tan^{-1}(1/x)$$

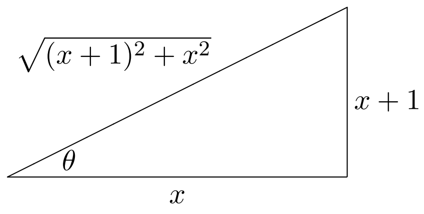
Composing a trig function and an inverse trig function.
The easiest way is to draw a right triangle diagram. For example: to compute $\sin\left(\cos^{-1}(x/3)\right)$, we draw a right triangle and mark an angle θ, which represents $\cos^{-1}(x/3)$. Then we make the adjacent side x and the hypoteneuse 3. By the pythagorean theorem, the opposite side is $\sqrt{9-x^2}$. Then,

$$\sin\theta = \frac{\text{opposite}}{\text{hypoteneuse}} = \frac{\sqrt{9-x^2}}{3}.$$



Another example: to compute $\sec\left(\tan^{-1}\left(\frac{x+1}{x}\right)\right)$ we use the triangle below, then we do

$$\sec\theta = \frac{\text{hypoteneuse}}{\text{adjacent}} = \frac{\sqrt{(x+1)^2+x^2}}{x}.$$



Completing the square.
We want to write a polynomial like ax^2+bx+c in the form $a(x+d)^2+e$. The formula is

$$d = \frac{b}{2a} \qquad e = c - \frac{b^2}{4a}.$$

Example: We can write x^2+4x+1 as $(x+2)^2-3$. It might help to think of it like this:

$$x^2+4x+1 = x^2+4x+(4-4)+1 = (x^2+4x+4)-3 = (x+2)^2-3.$$